CURSO: I BACHILLERATO

ASIGNATURA: Matemáticas NM

FECHA: Lunes 17 y martes 18 de agosto de 2015

TEMA: Identidades Trigonométricas.

OBJETIVOS:

✓ Simplificar expresiones trigonométricas.

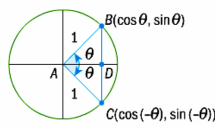
ACTIVIDADES A DESARROLLAR

ACTIVIDAD 01 (20 Minutos)

Docente demuestra la identidad del coseno y del seno del ángulo doble, usando como referencia el texto Mathematics Standard Level. Páginas 457 y 458. (Haga referencia al círculo unitario)

Double-angle identity for cosine

The diagram shows the angles θ and $-\theta$ drawn in standard position in the unit circle.



The length of segment CD is equal to the length of segment BD, and we have $BD = CD = \sin\theta$.

$$BC = BD + CD, \text{ so } BC = 2\sin\theta. \qquad [1]$$

We can see that $\angle BAC = 2\theta$. The length of segment BC can be found using the cosine rule in ΔABC:

$$BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos(2\theta)$$

$$BC^2 = 1^2 + 1^2 - 2(1)(1)\cos(2\theta) = 2 - 2\cos(2\theta)$$

$$BC = \sqrt{2 - 2\cos(2\theta)} \qquad [2]$$

Now we have two expressions for BC.

If we put [1] and [2] equal, we find

$$2\sin\theta = \sqrt{2 - 2\cos(2\theta)}.$$

Squaring both sides gives us $4\sin^2\theta = 2 - 2\cos(2\theta)$.
Rearranging this equation gives us $2\cos(2\theta) = 2 - 4\sin^2\theta$.
Finally, we divide by 2 to get $\cos(2\theta) = 1 - 2\sin^2\theta$.

→ The equation $\cos(2\theta) = 1 - 2\sin 2\theta$ is an **identity**, as it is true for all values of θ.

We will use this identity to help us find other identities.

We know $\sin^2\theta + \cos^2\theta = 1$, so $\sin^2\theta = 1 - \cos^2\theta$.
Using substitution, we get $\cos(2\theta) = 1 - 2(1 - \cos^2\theta)$.
Rearranging this equation gives us

$$\cos(2\theta) = 2\cos^2\theta - 1.$$

We can substitute $\sin^2\theta + \cos^2\theta = 1$ into this equation to get $2\cos(2\theta) = 2\cos^2\theta - (\sin^2\theta + \cos^2\theta)$, which gives us

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta.$$

The three equations we have just found are:

→ The **double-angle identities** for cosine:
$$\begin{aligned}\cos(2\theta) &= 1 - 2\sin^2\theta\\ &= 2\cos^2\theta - 1\\ &= \cos^2\theta - \sin^2\theta\end{aligned}$$

Double-angle identity for sine

Now we will find a double-angle identity for sine.

We know that $\sin^2(2\theta) + \cos^2(2\theta) = 1$, so

$$\cos^2(2\theta) = 1 - \sin^2(2\theta). \qquad [1]$$

From the double-angle identity for cosine,

$$\cos(2\theta) = 1 - 2\sin^2\theta$$

$$\cos^2(2\theta) = (1 - 2\sin^2\theta)^2 \qquad [2]$$

$$1 - \sin^2(2\theta) = (1 - 2\sin^2\theta)^2$$

Equate [1] and [2]

$$1 - \sin^2(2\theta) = 1 - 4\sin^2\theta + 4\sin^4\theta$$

$$4\sin^2\theta - 4\sin^4\theta = \sin^2(2\theta)$$

$$4\sin^2\theta(1 - \sin^2\theta) = \sin^2(2\theta)$$

$1 - \sin^2\theta = \cos^2\theta$

$4\sin^2\theta\cos^2\theta = \sin^2(2\theta)$

$2\sin\theta\cos\theta = \sin(2\theta)$

Take square roots of both sides

→ The double-angle identity for sine is $\sin(2\theta) = 2\sin\theta\cos\theta$

ACTIVIDAD 02. (20 Minutos)

Resuelva los ejemplos 15 y 16 del libro Mathematics for the International Student SL. Página 269.

Example 15 Self Tutor

Given that $\sin\alpha = \frac{3}{5}$ and $\cos\alpha = -\frac{4}{5}$ find:

a $\sin 2\alpha$ b $\cos 2\alpha$

a
$$\begin{aligned} &\sin 2\alpha \\ &= 2\sin\alpha\cos\alpha \\ &= 2(\tfrac{3}{5})(-\tfrac{4}{5}) \\ &= -\tfrac{24}{25} \end{aligned}$$

b
$$\begin{aligned} &\cos 2\alpha \\ &= \cos^2\alpha - \sin^2\alpha \\ &= (-\tfrac{4}{5})^2 - (\tfrac{3}{5})^2 \\ &= \tfrac{7}{25} \end{aligned}$$

Example 16 Self Tutor

If $\sin\alpha = \frac{5}{13}$ where $\frac{\pi}{2} < \alpha < \pi$, find the exact value of $\sin 2\alpha$.

α is in quadrant 2, so $\cos\alpha$ is negative.

Now $\cos^2\alpha + \sin^2\alpha = 1$

$\therefore \cos^2\alpha + \frac{25}{169} = 1$

$\therefore \cos^2\alpha = \frac{144}{169}$

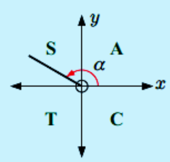
$\therefore \cos\alpha = \pm\frac{12}{13}$

$\therefore \cos\alpha = -\frac{12}{13}$

But $\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\therefore \sin 2\alpha = 2(\frac{5}{13})(-\frac{12}{13})$

$= -\frac{120}{169}$



ACTIVIDAD 03. (40 Minutos)

Los estudiantes resuelven en la pizarra, los ejercicios 1 a 4 del libro Mathematics for the International Student SL. Página 269.

EXERCISE 11D

1 If $\sin\theta = \frac{4}{5}$ and $\cos\theta = \frac{3}{5}$ find the exact values of:

a $\sin 2\theta$ b $\cos 2\theta$ c $\tan 2\theta$

2 a If $\cos A = \frac{1}{3}$, find $\cos 2A$. b If $\sin\phi = -\frac{2}{3}$, find $\cos 2\phi$.

3 If $\sin\alpha = -\frac{2}{3}$ where $\pi < \alpha < \frac{3\pi}{2}$, find the exact value of:

a $\cos\alpha$ b $\sin 2\alpha$

4 If $\cos\beta = \frac{2}{5}$ where $270° < \beta < 360°$, find the exact value of:

a $\sin\beta$ b $\sin 2\beta$